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Charmonium production at neutrino factories

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Abstract

At existing and planned neutrino factories (high energy and high intensity neutrino beam facilities) precision studies of QCD in neutrino-nucleon interactions are a realistic opportunity. We investigate charmonium production in fixed target neutrino experiments. We find that J/ψ production in neutrino-nucleon collision is dominated by the color octet 3S_1 NRQCD matrix element in a neutral current process, which is not accessible in photo or lepton production. Neutrino experiments at a future Muon Collider will acquire sufficient event rate to accurately measure color octet matrix element contributions. The currently running high energy neutrino experiments NOMAD and NuTeV could also observe several such events.

13.15+g, 13.85 Ni

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I. INTRODUCTION

One of the most promising future high energy facilities is the recently proposed Muon Collider. In order to facilitate a decision on whether and how it should be built, all the various uses it can be put to should be assessed. The highly collimated and intense neutrino beams unavoidably generated by muon decay provide a unique opportunity for precision studies of QCD and electroweak physics. An excellent example of such investigations is related to the ongoing issue of the validity of the Nonrelativistic QCD (NRQCD) expansion for charmonium states and the extraction of the so-called color octet matrix elements. The fact that heavy quarkonium represents a non-relativistic quantum-mechanical system significantly simplifies its theoretical studies. In particular, the presence of several important scales in the quarkonium system, M , Mv and Mv^2 ($\approx \Lambda_{QCD}$) where v is a small parameter (relative velocity of quarks in the quarkonium state) allows separation of physical effects occurring at different scales [1].

A large excess of prompt J/ψ 's and ψ' 's at the Tevatron over to the predictions of the color singlet model, i.e. the model which postulates that only quarks in a relative color singlet state can evolve into a charmonium, sparked both experimental and theoretical interest and resulted in the realization of the importance of contributions generated by the operators involving quark states in a relative color octet configuration. The emerging effective theory (NRQCD), systematically describing these processes, factorizes the charmonium production cross section in the form

$$\sigma(A + B \rightarrow H + X) = \sum_n \frac{F_n}{m_c^{d_n-4}} \langle 0 | \mathcal{O}_n^H | 0 \rangle, \quad (1)$$

where F_n are short-distance coefficients containing the perturbatively calculable hard physics of the production of a $[c\bar{c}]$ system at almost zero relative velocity (like $\gamma g \rightarrow c\bar{c}g$, $q\bar{q} \rightarrow c\bar{c}g$, etc.), expressed as a series in $\alpha_s(m_c)$. Here, the index n incorporates a spectral decomposition of the quarkonium state in terms of the quantum numbers $^{2S+1}L_J^{(color)}$ of the $[c\bar{c}]$ system, as well as the number of additional derivatives acting on the heavy quark fields. The essence of NRQCD is to organize the above expansion in powers of the heavy quark velocity v within the hadron, and it can be further generalized to include other heavy quarkonium-like systems, such as heavy hybrids [2,3]. Eq. (1) puts all nonperturbative long-distance information into the NRQCD matrix elements, which describe the evolution of the $[c\bar{c}]$ system into a charmonium plus soft hadrons – a process that cannot be calculated at present from first principles.

Several attempts have been made to determine these NRQCD matrix elements from various experiments. The processes involved are sensitive to various linear combinations of NRQCD matrix elements. The problem is aggravated by the usually very large theoretical uncertainties involved in these calculations (on the order of 50 – 100%), due to higher twist effects, uncalculated and/or incalculable higher order perturbative and nonperturbative contributions. In this situation any independent determination of these quantities should be welcome.

A major advantage of using the neutrino beam is that, at leading order in α_s , the spin structure of the νZ coupling selects a certain combination of octet operators. The largest contribution is from the one with the quantum numbers $^3S_1^{(8)}$. Of course, order of magnitude

measurements of the size of the matrix elements of this operator have already been performed for the J/ψ and ψ' , as well as for the χ_{cJ} states. The estimates of these matrix elements mostly come from Tevatron fits to hadroproduction cross sections for the J/ψ and χ_{cJ} and yield, with large theoretical errors [4],

$$\begin{aligned}\langle 0|\mathcal{O}_8^{J/\psi}(^3S_1)|0\rangle &\sim 0.01 \text{ GeV}^3, \quad \text{and} \\ \langle 0|\mathcal{O}_8^{\chi_{c0}}(^3S_1)|0\rangle &\sim 0.01 \text{ GeV}^3.\end{aligned}\tag{2}$$

These values are consistent, within a $\sim 50\%$ accuracy level, with the value found from Z decay at LEP [5] (the latter does not separate cascade and direct production, so the value of 0.019 GeV^3 is understandably larger than the one in Eq. (2)).

There are, however, large discrepancies between the Tevatron fits and the values of χ_{cJ} matrix elements obtained from B decays [6], and between various determinations of $\langle 0|\mathcal{O}_8^{J/\psi}(^3S_1)|0\rangle$ from the Tevatron fits. Clearly, new results from HERA leptonproduction experiments would not clarify the situation as at leading order the process $\gamma^*g \rightarrow [c\bar{c}]_8(^3S_1)$ is forbidden by parity conservation of strong interactions. In this situation other determinations are welcome and desired.

The present paper is an exploratory investigation of the main features of inclusive charmonium production in νN collisions. This process parallels J/ψ leptonproduction, in which case Fleming and Mehen [7] found that the $\mathcal{O}(\alpha_s^2)$ contribution to the total $eN \rightarrow e J/\psi X$ cross section is small compared to the color octet $\mathcal{O}(\alpha_s)$ contribution. A set of cuts, requiring an energetic gluon jet well separated from the J/ψ , enhances the $\mathcal{O}(\alpha_s^2)$ contributions, but then the color singlet contribution will dominate. These cuts, however, leave behind only a small part of the total cross section. We don't expect that either the difference in the spin structure or the $(m_W/m_Z)^4 \approx 0.6$ suppression of neutral current (NC) versus charged current (CC) events can change this picture, so that we feel justified to calculate only the $\mathcal{O}(\alpha_s)$ contributions. We will find, however, that while the leptonproduction of J/ψ is not sensitive to the $^3S_1^{(8)}$ matrix element, and measures one combination of $^1S_0^{(8)}$ and $^3P_J^{(8)}$, measuring the Q^2 distribution in our process allows for a determination of both the $^3S_1^{(8)}$ and the $^3P_J^{(8)}$ matrix elements. The difference is due to a difference in the spin structure of the Z and photon couplings.

The relative size of the $^3S_1^{(8)}$ and the $^3P_J^{(8)}$ contributions to the differential cross sections turns out to change drastically around $Q^2 \sim 10 \text{ GeV}^2$. This fact and the less steep decrease of the differential cross section (compared to leptonproduction) together allow an easy separation of the two contributing matrix elements.

We will find that the hard process at small Q^2 favors the $^3S_1^{(8)}$ contribution by a significant factor. In an experiment which cannot easily separate direct J/ψ production from a χ_{cJ} cascading down to a J/ψ (such is the situation at LEP, for example), these cascades enhance the observed cross section by a significant amount. In particular, given the estimates of Eq. (2), we find $\sigma(\nu N \rightarrow \chi_{c2} + X) \sim 5 \sigma(\nu N \rightarrow J/\psi + X)$! Noting that $BR(\chi_{c2} \rightarrow J/\psi \gamma) = 13.5\%$ we conclude that J/ψ production via this cascade mechanism is of the same order of magnitude as via the direct route. It is however easy to include the effect of these unresolved cascades into the formalism by simply replacing the actual $\langle 0|\mathcal{O}_8^{J/\psi}(^3S_1)|0\rangle$ matrix element with a $\langle 0|\hat{\mathcal{O}}_8^{J/\psi}(^3S_1)|0\rangle$ which takes into account this cascade factor [5]. The rate of χ_{cJ}

production can then easily be related to our $^3S_1^{(8)}$ contribution to the J/ψ production rate, because in that case there are no other competing matrix elements.

The large number of events expected at the muon collider will certainly allow to study the entire spectrum of charmonium states. Due to the fact that the dominant $^1S_0^{(8)}$ matrix element of the h_c and the dominant $^3S_1^{(8)}$ of the χ_{cJ} are related by heavy quark symmetry, our results can be trivially translated to a prediction of these rates. The only relevant matrix element above is measurable in any other cascade J/ψ production process.

In the present study we also discuss what we can learn about these matrix elements at existing neutrino facilities. As we show below, the mere event of the detection of charmonium states in current experiments (NOMAD or NuTeV) would imply the presence of the color-octet structures predicted by the NRQCD factorization formalism. The main question to be addressed here is the smallness of the event number. Since the detectors used in current neutrino experiments are optimized for the observation of neutrino-related phenomena (such as neutrino oscillations) and not for charmonium detection, we shall concentrate on J/ψ production, which has the cleanest experimental signature of all charmonium states. However cascade processes should also be included, helping to increase the event number. We will find that both at NOMAD and NuTeV, the event rate is on the verge of observability.

The rest of the paper is organized as follows. In Sec. II we explain the velocity scaling of the NRQCD matrix elements that we use and provide analytical formulas for the structure functions F_n . We discuss the numerical results for the Muon Collider, NOMAD and NuTeV in detail in Sec. III and offer concluding remarks in the Conclusions.

II. THE CALCULATION

At leading order, $\mathcal{O}(\alpha_s)$, only neutral current gZ interactions contribute to charmonium production and Fig. 1 shows the relevant Feynman diagrams. At $\mathcal{O}(\alpha_s^2)$, in addition to the color singlet contribution, one may expect some enhancement from charged current interactions due to the difference in the couplings and the propagators, but we do not expect that this could drastically change the cross section to invalidate our order-of-magnitude estimates.



FIG. 1. The leading $\mathcal{O}(\alpha_s)$ graphs. Note the absence of any W exchange graphs.

Before proceeding with the calculation, let us briefly review the velocity counting rules of the charmonium production matrix elements which will be used below. The charmonium production matrix elements are defined in NRQCD as

$$\langle 0|\mathcal{O}_n^H|0\rangle = \sum_X \sum_{m_J} \langle 0|\mathcal{K}_n|H_{m_J} + X\rangle \langle H_{m_J} + X|\mathcal{K}'_n|0\rangle \quad (3)$$

where the operators $\mathcal{K}_n^{(\prime)}$ are bilinear in the heavy quark fields. (m_J is the charmonium spin z -component.) Each matrix element is proportional to a power of the relative velocity v of the heavy quarks. Each spatial derivative acting on a heavy quark field introduces one power of v . The nonperturbative evolution of the $c\bar{c}$ system proceeds through multipole radiation of soft gluons, which introduce additional powers of the velocity. For the J/ψ and ψ' , the leading matrix element is at $m_c^3 v^3$,

$$\langle 0|\mathcal{O}_1^{J/\psi}({}^3S_1)|0\rangle = \langle 0|\chi^\dagger \sigma \psi|J/\psi + X\rangle \langle J/\psi + X|\psi^\dagger \sigma \chi|0\rangle, \quad (4)$$

which clearly selects the dominant Fock state. We observe, however, that this matrix element corresponds to a $c\bar{c}$ system in a color singlet state, which can be produced only at subleading order, $\mathcal{O}(\alpha_s^2)$. Therefore, its contribution will be smaller than that of the velocity-subleading color octet configuration, such as

$$\begin{aligned} \langle 0|\mathcal{O}_8^{J/\psi}({}^3S_1)|0\rangle &= \langle 0|\chi^\dagger \sigma T^a \psi|J/\psi + X\rangle \langle J/\psi + X|\psi^\dagger \sigma T^a \chi|0\rangle, \\ \langle 0|\mathcal{O}_8^{J/\psi}({}^1S_0)|0\rangle &= \langle 0|\chi^\dagger T^a \psi|J/\psi + X\rangle \langle J/\psi + X|\psi^\dagger T^a \chi|0\rangle, \\ \langle 0|\mathcal{O}_8^{J/\psi}({}^3P_J)|0\rangle &= \langle 0|\chi^\dagger \left[-\frac{i}{2}D^{\{i}\sigma^k\}}\right] T^a \psi|J/\psi + X\rangle \langle J/\psi + X|\psi^\dagger \left[-\frac{i}{2}D^{\{i}\sigma^k\}}\right] T^a \chi|0\rangle, \\ \langle 0|\mathcal{O}_8^{J/\psi}({}^1P_0)|0\rangle &= \langle 0|\chi^\dagger \left[-\frac{i}{2}\mathbf{D}\right] T^a \psi|J/\psi + X\rangle \langle J/\psi + X|\psi^\dagger \left[-\frac{i}{2}\mathbf{D}\right] T^a \chi|0\rangle, \end{aligned} \quad (5)$$

where $\{\dots\}$ represents scalar, vector or traceless symmetric tensor contraction of indices. Five of these configurations, namely those with the quantum numbers ${}^1S_0^{(8)}$, ${}^3S_1^{(8)}$, ${}^3P_J^{(8)}$ for $J = 0, 1, 2$, scale as v^7 . The ${}^1P_0^{(8)}$ matrix element is negligible, of the order $\mathcal{O}(v^{11})$, and will be neglected hereafter.

In the case of the p-wave states χ_{cJ} we encounter a similar situation. The leading matrix elements are again suppressed by one factor of α_s , and are color singlets at $m_c^5 v^5$. They project out the dominant p-wave $c\bar{c}$ combination

$$\begin{aligned} \langle 0|\mathcal{O}_1^{\chi_{c0}}({}^3P_0)|0\rangle &= \frac{1}{3} \langle 0|\chi^\dagger \left[-\frac{i}{2}\mathbf{D} \cdot \sigma\right] \psi|\chi_{c0} + X\rangle \langle \chi_{c0} + X|\psi^\dagger \left[-\frac{i}{2}\mathbf{D} \cdot \sigma\right] \chi|0\rangle \\ \langle 0|\mathcal{O}_1^{\chi_{c1}}({}^3P_1)|0\rangle &= \frac{1}{2} \langle 0|\chi^\dagger \left[-\frac{i}{2}\mathbf{D} \times \sigma\right] \psi|\chi_{c1} + X\rangle \langle \chi_{c1} + X|\psi^\dagger \left[-\frac{i}{2}\mathbf{D} \times \sigma\right] \chi|0\rangle \\ \langle 0|\mathcal{O}_1^{\chi_{c2}}({}^3P_2)|0\rangle &= \langle 0|\chi^\dagger \left[-\frac{i}{2}D^{(i}\sigma^k)}\right] \psi|\chi_{c2} + X\rangle \langle \chi_{c2} + X|\psi^\dagger \left[-\frac{i}{2}D^{(i}\sigma^k)}\right] \chi|0\rangle. \end{aligned} \quad (6)$$

The leading matrix elements in the velocity expansion are ${}^3P_J^{(1)} \sim m_c^5 v^5$ and ${}^3S_1^{(8)} \sim m_c^3 v^5$, but the ${}^3S_1^{(8)}$ configuration is produced at leading order in α_s :

$$\langle 0|\mathcal{O}_8^{\chi_{cJ}}({}^3S_1)|0\rangle = \langle 0|\chi^\dagger \sigma T^a \psi|\chi_{cJ} + X\rangle \langle \chi_{cJ} + X|\psi^\dagger \sigma T^a \chi|0\rangle. \quad (7)$$

The velocity counting rules in NRQCD require, for the most interesting case of the J/ψ , to calculate the contributions of the $\mathcal{O}(v^7)$ matrix elements, ${}^1S_0^{(8)}$, ${}^3S_1^{(8)}$, ${}^3P_J^{(8)}$ (heavy quark

symmetry requires ${}^3P_J^{(8)} = (2J+1) {}^3P_0^{(8)}$. We note that the same structure functions can be used to evaluate all contributions to χ_c , for which ${}^3S_1^{(8)}$ is leading, of order $\mathcal{O}(v^5)$, and to h_c , for which ${}^1S_0^{(8)}$ is leading, also of order $\mathcal{O}(v^5)$. We do not calculate the hard-to-identify η_c , where the uncalculated ${}^1P_0^{(8)}$ would also contribute at leading order (this matrix element is completely unimportant for the J/ψ , as it is suppressed as $\mathcal{O}(v^{11})$ in that case). We also note that parity and J conservation forbids all interference terms in Eq. (8).

The calculation is quite straightforward and we quote the result in the form of the following structure functions $h_n(Q^2)$:

$$\frac{d\sigma(s, Q^2)}{dQ^2} = \frac{\pi^2 \alpha^2 \alpha_s}{3 \sin^4 2\theta_W} \frac{1}{(Q^2 + m_Z^2)^2} \times \sum_n \frac{\langle 0 | \mathcal{O}_n | 0 \rangle}{m_c^3} \int_{\frac{Q^2+4m_c^2}{s}}^1 dx f_{g/N}(x, Q^2) h_n(y, Q^2), \quad (8)$$

where s is the total invariant mass of the νN system, x is the momentum fraction of the incoming gluon and $-Q^2$ is the momentum-squared transferred from the leptonic system; $y = \frac{Q^2+4m_c^2}{sx}$. Because we are doing a tree level calculation, α_s does not run; in the actual calculation we will choose the educated guess of evaluating $\alpha_s \Rightarrow \alpha_s(Q^2 + 4m_c^2)$.

Now, our calculation gives

$$\begin{aligned} h_{{}_1S_0^{(8)}}(y, Q^2) &= (g_V^c)^2 \times 6 \frac{Q^2 m_c^2}{(Q^2 + 4m_c^2)^2} (y^2 - 2y + 2) \\ h_{{}_3S_1^{(8)}}(y, Q^2) &= (g_A^c)^2 \times 2m_c^2 \frac{Q^2(y^2 - 2y + 2) + 16(1-y)m_c^2}{(Q^2 + 4m_c^2)^2} \\ h_{{}_3P_0^{(8)}}(y, Q^2) &= (g_V^c)^2 \times 2Q^2 \frac{(Q^2 + 12m_c^2)^2}{(Q^2 + 4m_c^2)^4} (y^2 - 2y + 2) \\ h_{{}_3P_1^{(8)}}(y, Q^2) &= (g_V^c)^2 \times 4Q^4 \frac{Q^2(y^2 - 2y + 2) + 16(1-y)m_c^2}{(Q^2 + 4m_c^2)^4} \\ h_{{}_3P_2^{(8)}}(y, Q^2) &= (g_V^c)^2 \times \frac{4}{5} Q^2 \frac{(y^2 - 2y + 2)Q^4 + 48(1-y)Q^2m_c^2 + 96(y^2 - 2y + 2)m_c^4}{(Q^2 + 4m_c^2)^2} \end{aligned} \quad (9)$$

where $g_A^c = \frac{1}{2}$ and $g_V^c = \frac{1}{2} \left(1 - \frac{8}{3} \sin^2 \Theta_W\right)$ are the vector and axial couplings of the c -quark. We have checked that an appropriately modified version of these structure functions correctly reproduces the $e N \rightarrow e J/\psi X$ cross section in Ref. [7]. We immediately observe that the coupling constants favor the ${}^3S_1^{(8)}$ contribution, which is due to the large axial coupling (a similar contribution is, of course, absent in the case of J/ψ lepto and photoproduction). Indeed our numerical estimates will show that this matrix element dominates the total cross section, and also the differential cross section unless $Q^2 \gg m_c^2$. At large Q^2 , the relative Q^4 enhancement of the P -wave structure functions makes them dominant.

These structure functions should eventually be incorporated in the specific Monte Carlo generators built for each particular detector. Our numerical estimates presented in the following chapters ignore the fine details of the experiments and should be understood as a preliminary feasibility study.

III. NUMERICAL RESULTS

Due to its clean experimental signature, one of the most important charmonium production processes is J/ψ production. We have calculated the total cross section and the differential Q^2 distribution of J/ψ 's at various experiments, both currently running and proposed. For lack of knowledge of the size of the NRQCD matrix elements we set them equal to a reasonable “baseline size” value, defined as

$$\langle 0 | \mathcal{O}_8(^1S_0) | 0 \rangle_{baseline} = \langle 0 | \mathcal{O}_8(^3S_1) | 0 \rangle_{baseline} = \frac{\langle 0 | \mathcal{O}_8(^3P_J) | 0 \rangle_{baseline}}{(2J+1)m_c^2} = 10^{-2} GeV^3 \quad (10)$$

These values satisfy the restrictions imposed by heavy quark symmetry and are compatible with the existing extractions [4,6,8]. The results below were found using MRST parton distribution functions (PDF) [9], substituting $m_c = 1.35 GeV$ for the charm quark mass and imposing a cut $Q^2 > (1.2 GeV)^2$ in the calculation of the total cross section.

The energy dependence of the total cross section, taken at various representative incident neutrino energies, is shown in Table I. We find that the total cross section is very sensitive to the neutrino beam energy E_ν . The origin of this strong dependence can be traced to the fact that the main contribution to the integral in Eq. (8) comes from the smallest available $x \approx \frac{Q^2 + 4m_c^2}{s}$, so that larger s probes a smaller region of x where the gluon PDF's are enhanced. It becomes obvious that in order to have a reasonably large event number, the neutrino beams with the highest possible energy should be employed.

$E_\nu[GeV]$	7.5	25	120	450
$\sigma[nb]$	7.8×10^{-13}	6.9×10^{-10}	1.3×10^{-8}	5.5×10^{-8}

TABLE I. Total cross sections for the J/ψ production in $\nu N \rightarrow J/\psi X$ for various representative incident neutrino energies. The values are taken to correspond to the minimal, average and maximum neutrino beam energy at NOMAD and maximum available neutrino beam energy at NuTeV respectively.

We have calculated the total cross section for the two presently running high energy neutrino experiments, NOMAD at CERN and NuTeV at Fermilab, as well as for the much-discussed high energy muon collider. Table II shows the results for each term in the spectral decomposition. The values support our previous expectations that the $^3S_1^{(8)}$ contribution dominates the total cross section. Note that these numbers refer to direct J/ψ production; a cascade mechanism involving χ_{cJ} decay into J/ψ can be easily calculated from our $^3S_1^{(8)}$ contribution and its ratio to direct $^3S_1^{(8)}$ production is universal and can be taken from other experiments.

We should emphasize that due to the expected small number of events the question of background suppression becomes very important. In particular, in the case of the currently running experiments, NOMAD and NuTeV, the leptonic decay channel of J/ψ becomes virtually the only possibility of detecting the produced J/ψ 's. This makes electromagnetic lepton pair production a very important source of background [10].

Spectral decomp.	NOMAD		NuTeV		FMC-Ring		FMC-RLA3	
	$\sigma [10^{-9} \text{ nb}]$	$10^6 \frac{\sigma}{\sigma^{tot}}$	$\sigma [10^{-9} \text{ nb}]$	$10^6 \frac{\sigma}{\sigma^{tot}}$	$\sigma [10^{-9} \text{ nb}]$	$10^6 \frac{\sigma}{\sigma^{tot}}$	$\sigma [10^{-9} \text{ nb}]$	$10^6 \frac{\sigma}{\sigma^{tot}}$
$^1S_0(8)$	0.086	0.0131	1.59	1.94	2.01	3.89	1.54	2.30
$^3S_1(8)$	0.552	0.0842	9.14	11.2	11.5	22.3	8.97	13.4
$^3P_0(8)$	0.129	0.0195	1.92	2.34	2.40	4.65	1.89	2.82
$^3P_1(8)$	0.082	0.0124	2.06	2.52	2.64	5.10	1.96	2.94
$^3P_2(8)$	0.166	0.0253	2.71	3.30	3.42	6.61	2.65	3.97
SUM	1.01	0.154	17.4	21.3	22.0	42.5	17.0	25.4
Total	$\sigma^{tot} = 6.56 \text{ pb}$		$\sigma^{tot} = 0.82 \text{ pb}$		$\sigma^{tot} = 0.52 \text{ pb}$		$\sigma^{tot} = 0.67 \text{ pb}$	

TABLE II. The $\nu N \rightarrow J/\psi X$ cross section contribution from each matrix element (set equal to its baseline value) in the four discussed experiments, folding in the neutrino energy distributions. Here, σ^{tot} refers to the total deep inelastic cross section, $\sigma^{tot} = \sigma_{CC} + \sigma_{NC}$.

Another important issue is diffractive J/ψ production. It is difficult to separate, both theoretically and experimentally, diffractive and color octet contributions. This is due to the fact that both of these processes contribute at $z \equiv (P_N \cdot P_\psi)/(P_N \cdot q) \sim 1$. In principle, the absence of a rapidity gap in the color-octet NRQCD process may help [7]. In addition, a perturbative QCD calculation of the diffractive lepton production [11] suggests that this process falls off at high Q^2 as $\sim 1/Q^6$. We expect this behavior to hold for the case of diffractive “neutrinoproduction” as well. As indicated in our calculation, the color octet contribution falls off only as $\sim 1/Q^4$ which makes the diffractive process negligible at sufficiently high Q^2 . Figs. 2 and 4 show the Q^2 distribution of the events. As expected, these decrease quickly with increasing Q^2 . The decrease, however, is not so drastic as not to let us use higher Q^2 cut up to $\mathcal{O}(10 \text{ GeV}^2)$ which may also allow to reduce the nonperturbative background.

Now we turn to the discussion of some of the presently running and planned neutrino experiments.

A. The muon collider

A proposal to build a high energy $\mu^+\mu^-$ storage ring has recently drawn much attention. Although seemingly a sideline, luminous neutrino beams are unavoidably generated by the decaying muons, and provide an ideal environment for charmonium generation. Such machines comprise both necessary ingredients, high luminosity and high neutrino beam energy, that are needed for detailed studies of charmonium production.

The proposed First Muon Collider [12] would use $2 \times 250 \text{ GeV}$ muons, boosted in linear accelerators. These muons and antimuons decay into the ν_μ ’s (or $\bar{\nu}_\mu$ ’s) in both final booster (called RLA3) and in the accelerator ring. A straight section of the ring was proposed in order to achieve a highly collimated neutrino beam. The resulting neutrino spectra that have been calculated by Harris and McFarland [13] peak in the $150 - 200 \text{ GeV}$ region. With a representative set of parameters $\mathcal{O}(10^6)$ DIS events/(g/cm² year) are expected. This high number of events allows a measurement of the partial distributions without the need for extremely costly dedicated detectors. With the cross sections given in Table II, this

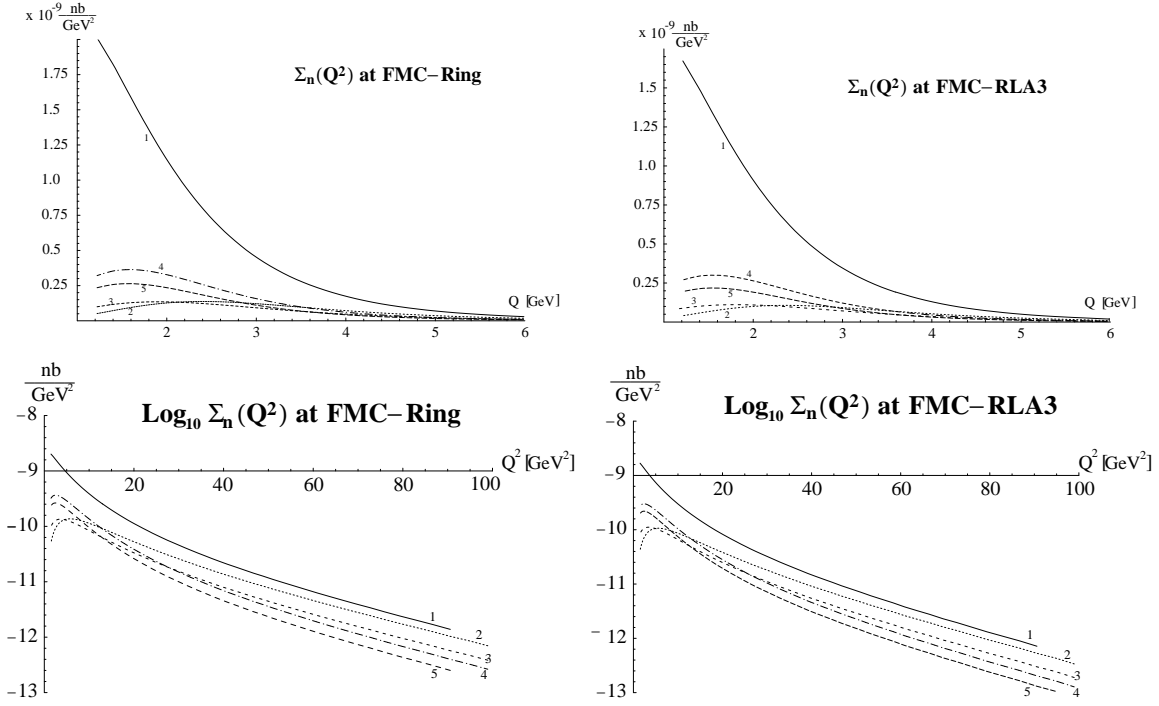


FIG. 2. The Q^2 distribution $\Sigma_n[\frac{nb}{GeV^2}]$ of the J/ψ events at the FMC for baseline-sized matrix elements: $d\sigma_n = \mu_n d\Sigma_n$ when $\langle \mathcal{O}_n \rangle = \mu_n \langle \mathcal{O}_n \rangle_{baseline}$. The solid lines (1) correspond to ${}^3S_1^{(8)}$, the dotted lines (2) – ${}^3P_1^{(8)}$, the short-dashed lines (3) – ${}^1S_0^{(8)}$, the dash-dotted lines (4) – ${}^3P_2^{(8)}$, and the long-dashed lines (5) are the ${}^3P_0^{(8)}$ contributions.

translates to 20 – 50 J/ψ events/(g/cm² year).

We will now sample the various suggestions for the neutrino detectors to get a rough idea of the yield and capabilities. In all cases we look at a detector diameter of 40 cm, in which almost all of the neutrino beam fits.

As an obvious minimal suggestion, we might look at a 30 cm thick, table-top sized detector [13], containing approximately 50 kg of water-density target. We estimate a yield of 1600 J/ψ events/year, providing just enough statistics for a crude estimate of the octet matrix elements. By contrast, a light target considered e.g. in [14] (a 1 m thick liquid hydrogen detector) would only produce approximately 100 J/ψ events/year, not much better than presently running experiments.

The best answer may be the “general purpose detector” suggested by B. King [15]. This detector consists of a one meter long stack of silicon CCD tracking planes, providing ~ 50 g/cm² density. In such a detector an estimated 3,000 J/ψ events/year would occur and the high efficiency and precise reconstruction capabilities should allow for disentangling the contribution from the color octet ${}^3S_1^{(8)}$ and ${}^3P_J^{(8)}$ matrix elements, through the measurement of the differential Q^2 distributions. Note that an increase in the length of the straight section of the storage ring provides a relatively cheap way of increasing the luminosity of the neutrino beam, resulting in better statistics. Ref. [15] actually considers a 200 m long straight section,

gaining a factor of 20 over our estimates.

Another option is a conventional fixed target type heavy detector [13]. A representative example of a 2 ton iron calorimeter 1.6 m long, would see very high event rates (approximately 70,000 J/ψ events/year before cuts and detection efficiencies are included). If background difficulties can be overcome, this option provides an excellent opportunity to study all differential rate distributions with precise statistics.

As Fig. 2 shows, the differential cross section quickly decreases with Q^2 . However, the decrease is not as fast as in the leptonproduction case. Comparing the Q^2 behavior of formula for the $^3S_1^{(8)}$ contribution to Eq. (3) in Ref. [7], we find

$$\left(\frac{d\sigma}{dQ^2}\right)_{\nu N} \propto \left(\frac{d\sigma}{dQ^2}\right)_{eN} \times \frac{Q^2(Q^2 + \beta m_c^2)}{m_Z^4} \quad (11)$$

where $\beta \sim 4 - 10$ is slightly Q^2 -dependent. We have checked that our curves actually reproduce this relationship with $\beta \approx 6$. The upshot is that the absence of a photon propagator results in a much wider tail of the Q^2 distribution than in leptonproduction, allowing for better discrimination between S - and P -wave contributions.

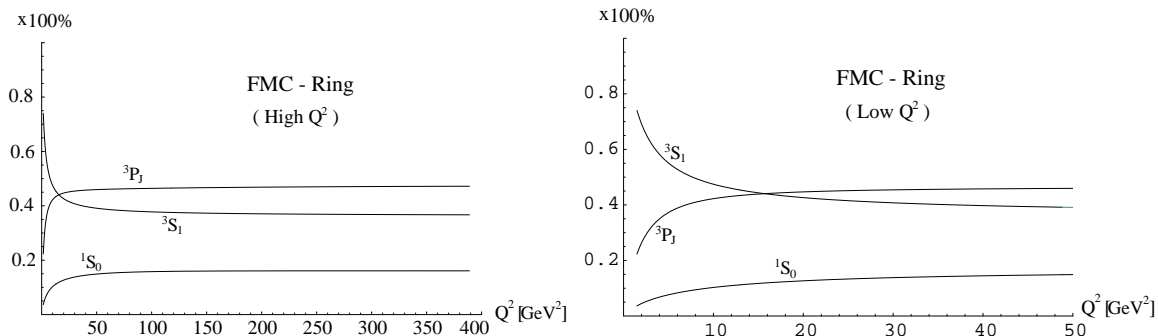


FIG. 3. The contributions of the various matrix elements as a function of Q^2 for FMC-Ring, with baseline-sized matrix elements. Observe that the contributing linear combination of matrix elements drastically changes with Q^2 within a region where the rate is still sizable, so one can realistically extract both the $^3S_1^{(8)}$ and the $^3P_J^{(8)}$ matrix elements even without requiring exceedingly high statistics.

Fig. 3 shows the ratio of the contributions from these matrix elements as a function of Q^2 . At low Q^2 , where the contribution into the cross section is large, the $^3S_1^{(8)}$ dominates but a determination of the differential cross section in the range $\sim 2 \text{ GeV} < Q < 4 \text{ GeV}$, where the event number is still sizable, would allow a reliable extraction of both $^3S_1^{(8)}$ and $^3P_J^{(8)}$ matrix elements. Higher Q^2 's will only measure a linear combination of these, as the ratio in Fig. 3 becomes Q^2 -independent.

Given the above calculation, it is easy to estimate the production rate of other charmonium states, for instance η_c . Using heavy quark symmetry, it is possible to relate, at leading order in v^2 , the leading $\mathcal{O}(v^7)$ matrix elements of η_c , $^1S_0^{(8)}$, $^3S_1^{(8)}$, $^1P_1^{(8)}$ to the corresponding J/ψ matrix elements. A simple calculation shows, using the data in Table II with baseline-sized matrix elements, that the η_c yield will be one third of the J/ψ 's. This fact and the

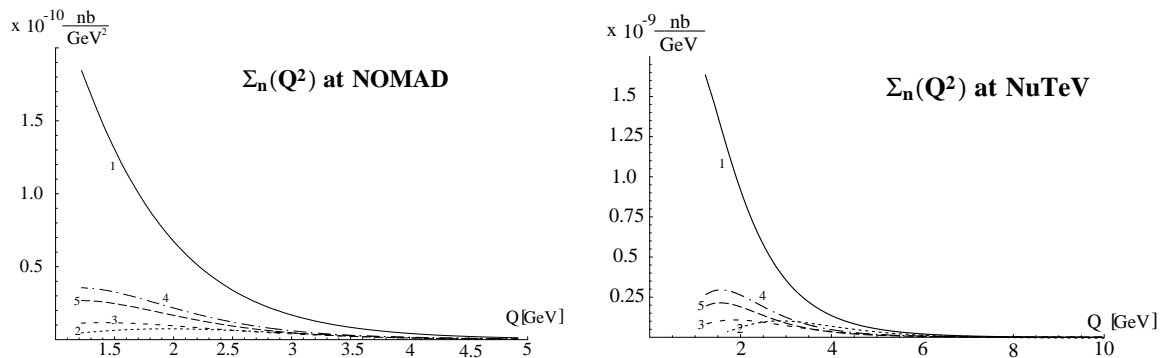


FIG. 4. The Q^2 distribution $\Sigma_n[\frac{nb}{GeV^2}]$ of the J/ψ events at NOMAD and NuTeV. The solid lines (1) correspond to $^3S_1^{(8)}$, the dotted lines (2) – $^3P_1^{(8)}$, the short-dashed lines (3) – $^1S_0^{(8)}$, the dash-dotted lines (4) – $^3P_2^{(8)}$, and the long-dashed lines (5) are the $^3P_0^{(8)}$ contributions.

absence of a clear signature will make η_c quite elusive in this experiment. A similar remark holds for the h_c , whose only contributing matrix element, the $\mathcal{O}(v^5)$ level $^1S_0^{(8)}$, is related to the $^3S_1^{(8)}$ matrix elements of the χ_{cJ} 's.

B. NOMAD

The NOMAD detector at CERN uses the CERN SPS neutrino beam, mainly designed for detecting neutrino oscillations. The average neutrino energy is small, 24 GeV, and, even with the huge mass of the detector, the event number is small. One would expect approximately 2 – 3 events per year in the main detector with its fiducial mass of 2.7 tons. The situation is somewhat better in the Front Calorimeter, where such a search is underway, simply because of the larger mass of the detector.¹ With a mass of 17.7 tons, and the detector characteristics given in Ref. [16], the yield becomes 14 – 15 J/ψ events/year, which must be multiplied by the decay ratios and detector efficiencies to find the number of observable events. Given the inaccuracies of our calculation and the crude estimate of the size of the matrix element, it does not seem impossible that several such events would be seen.

C. NuTeV

Another high energy neutrino oscillation experiment has a chance of probing, as a byproduct, our process. The larger average neutrino energy at the Fermilab experiment, compared to CERN, results in approximately twenty times larger cross section, compensating for the smaller size of the detector. The Fermilab experiment NuTeV detected, during its lifetime

¹The authors thank Kai Zuber for drawing their attention to this point.

of one year, about 1.3 million DIS events. Using the numbers in Table II, this would imply 28 J/ψ events. This is again on the verge of observability.

IV. CONCLUSIONS

In conclusion we note that an analogous calculation can be done for the $b\bar{b}$ system. In that case, however, the α_s suppression of the ratio of the color singlet vs. color octet contributions is compensated by the smallness of the heavy quark velocity. Indeed, we expect

$$\frac{(singlet/octet)_{b\bar{b}}}{(singlet/octet)_{c\bar{c}}} \sim \frac{(\alpha_s/v^4)_{b\bar{b}}}{(\alpha_s/v^4)_{c\bar{c}}} \sim 6 \quad \text{because} \quad \frac{\alpha_s(m_\Upsilon)}{\alpha_s(m_{J/\psi})} \sim \frac{2}{3}, \quad \text{and} \quad \left(\frac{v_c^2}{v_b^2}\right)^2 \sim \left(\frac{0.3}{0.1}\right)^2 \sim 9. \quad (12)$$

(Recall that in J/ψ leptonproduction the color singlet contribution was suppressed by $\sim 10\%$.) As a consequence, our tree level calculation is unable to accurately predict the dominant Υ production rate. However, because all numerical factors in the color octet contribution are smaller for the Υ than for the J/ψ (including the size of the phase space), we can at least exclude the possibility of Υ production at presently running experiments. In order to make a meaningful estimate for the muon collider, one needs to perform much more involved $\mathcal{O}(\alpha_s^2)$ calculations.

To summarize, we have investigated charmonium production at some of the currently running (NOMAD and NuTeV) and proposed (First Muon Collider) neutrino factories. We found that the J/ψ production cross section is dominated by the contribution from the matrix element of the color octet $^3S_1^{(8)}$ operator, which is not accessible in photo and leptonproduction experiments. Our exploratory study shows that the neutrino beam from the $\mu^+\mu^-$ collider can be successfully used to accurately extract the color octet $^3S_1^{(8)}$ and $^3P_J^{(8)}$ matrix elements of the J/ψ . This measurement is complementary to leptonproduction, where 3S_0 and 3P_J contribute. In order to achieve an accurate extraction, however, a detailed investigation is necessary, including the $\mathcal{O}(\alpha_s^2)$ contributions. A significant deviation of the measured values of the nonperturbative charmonium matrix elements extracted from high energy νp and $p\bar{p}$ experiments could be a sign of intrinsic charm in the proton.

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